

On the Resistance of Air.

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The great mass of work, both theoretical and experimental, which has been done on this subject may be divided into two classes, one of which comprises all the results having reference to such moderate velocities as can be obtained with artificially produced currents (*i.e.*, up to about 100 f.s.), while the other is confined to velocities about and exceeding the velocity of sound. Between these two there is a gap in the experimental record extending over a range of nearly 1000 f.s.

I am not aware that any attempt has been made to connect theoretically the experimental results at high and low velocities, and it is the object of this paper to find an expression which will represent the resistance generally.

The expression which is arrived at does not pretend to be anything but an arbitrary formula, but it is, I think, useful as separating the resistance into its component parts, and also as indicating the class of experiment which is needed to improve this provisional quantification of each of them.

At low velocities it is known that the resistance varies as the square of the velocity (as it would at all velocities in an incompressible fluid), but two distinct cases may be noticed depending on the shape of the resisting body. If this has what, in a ship, would be called a fine run, that is, if the “after” part of the body tapers very gradually, so that the stream lines follow its contours, any resistance experienced by it is due to surface friction only. On the other hand, if the hinder part of the body is flat or tapers quickly, the stream lines leave the surface, and the body carries behind it a wake made up of a complex system of eddies whose formation requires a continuous expenditure of work.

The resistances considered in this paper are those of bodies which form a wake. One of the simplest cases is that of a plane moving in the direction of its normal. This, as a two-dimensional problem, has been solved—by Kirchhoff and by Lord Rayleigh—on the assumption that stream lines and wake are separated by a surface of discontinuity, the curvature of which is settled by the consideration that the pressure throughout the wake must be the same as the pressure in the distant parts of the fluid. The corresponding solution in three dimensions has not yet been effected.

The two-dimensional solution would apply accurately to the case of the

resistance of a long lamina in air moving slowly, if the space occupied by the theoretical wake and bounded by the surface of discontinuity was filled by a solid with a surface devoid of friction.

In the real case, however, the fluid wake is not at the pressure of the distant fluid, because, as shown in fig. 1, the eddies formed at the edge are

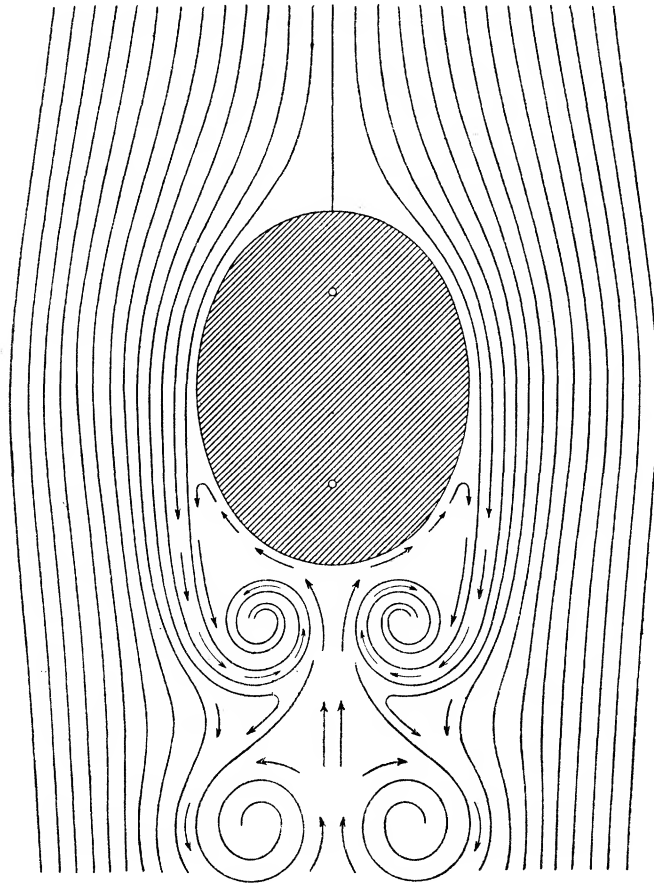


FIG. 1.

for ever wrapping up and drawing out from behind the plane part of the fluid which is found there. In consequence of this, fluid in the wake is not at rest relatively to the plane, but in its central part is flowing towards the plane to make good the loss of fluid abstracted by the eddies at its sides.

It must be noticed that the eddies in the wake are not of the nature of vortices in which each filament has the same velocity potential, but consist of a kind of sandwich-like structure, about half the fluid belonging to each

eddy being derived from the interior of the wake and half from the general stream outside. In two dimensions the formation of the eddies is a discontinuous process. In fig. 2, *a*, *b*, and *c*, illustrate their gradual growth, in (*a*) the eddy is just beginning to form at the edge of the plane, in (*c*) the growth

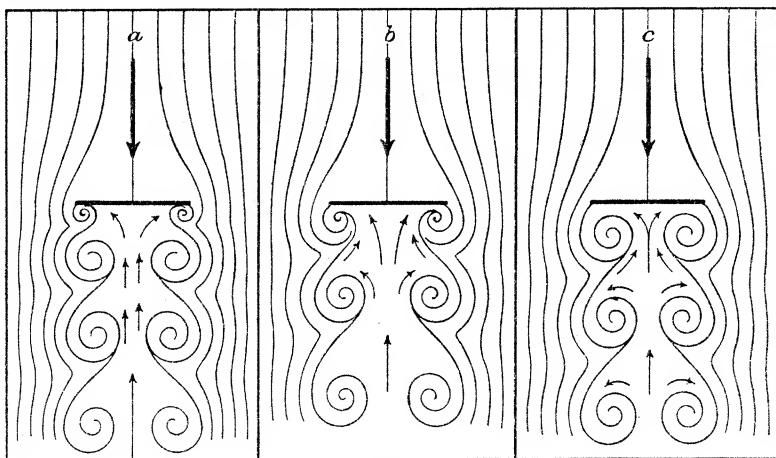


FIG. 2.

has proceeded until it impedes the forward flow of the wake which is necessary to feed it. When this stage is reached, the full-grown eddy breaks away and joins the procession of eddies forming the margin of the wake.

In three dimensions the eddies may be produced continuously, being in different stages of growth at different parts of the perimeter of the body behind which they are formed. In this case they either appear in the wake as a spiral* (see fig. 3), or, if the circumstances are such as to favour the inception of the eddy at some one point of the body, a current will be formed in the direction of the axis of the eddy.†

The rate at which eddies are produced has an important bearing on many natural phenomena.

In the two-dimensional problem the eddies may be formed symmetrically and simultaneously at the two edges, or alternately, in which latter case the wake consists of a series of alternate right-handed and left-handed eddies, fig. 4. If the wake-forming body is supported laterally, so that its natural period at right angles to the direction of the stream is t , then if the flow of the stream and dimensions of the body are such that the rate of eddy formation approaches t , the body will oscillate laterally with an amplitude increasing until the natural extinction balances the work done.

* Traces of the spiral may be seen in the shadow photograph, fig. 7.

† These phenomena are best examined by intermittence illumination, using a rate of intermittence slightly different from that of the eddy formation.

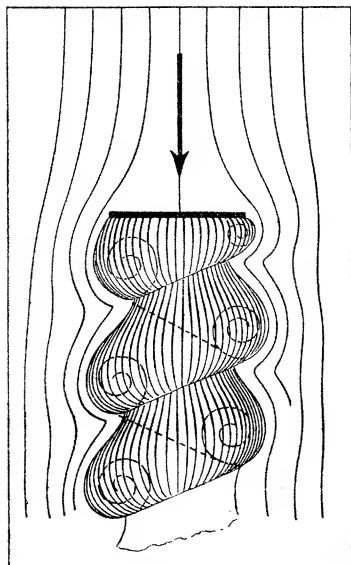


FIG. 3.

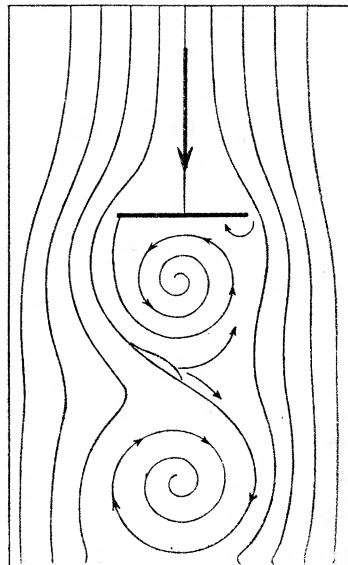


FIG. 4.

To this cause are due the vibrations of an *Æolian* harp string, and many other examples might be given.

The motion of the fluid in the wake is so complicated that it seems hopeless to treat it mathematically except in a statistical manner, and even for this data are at present wanting. It is known, however, that for low velocities in air the pressure on the hind surface of a plane is less than the general pressure in the fluid by about half the excess of pressure on the front surface.

As speed increases and approaches the velocity of sound, the negative pressure increases, because, apart from the question of supply to the eddies, the gas can only follow the plane at a reduced pressure, and when the velocity reaches 2400 f.s. about (in air) the negative pressure is equal to one atmosphere, and the hind surface is covered by a vacuum.

The negative pressure on the hind surface will be hardly affected by the shape of the front, hence, as one of the elements of resistance, we shall have a term which at first increases as the square of the velocity, grows rapidly as the velocity approaches that of sound, and at rather over 2400 f.s. becomes constant and equal to 1 atmosphere.

Next, as to the resistance of the head. I will suppose in the first place that the head is of such a shape that it would, in an incompressible fluid, get the full dynamic head due to the velocity over its whole surface.* (In an

* A plane fulfils this condition nearly, but not quite, as there is a small loss of pressure at the margins.

incompressible fluid the resistance of any other shape would be some constant fraction of this.)

Lord Rayleigh gives (in a paper on the Resistance of Fluids)* a formula for the head resistance of such a body as that just mentioned in an elastic gas (in which the pressure and density are connected by the equation $p/p_0 = (\rho/\rho_0)^\gamma$). This formula makes the resistance per unit area equal to the pressure required to cause a gas to flow from a vessel with the given velocity.

When we come to actually draw the curve which the formula represents, and compare it with the results of experiment, it will be seen that the two do not agree, and this not in the magnitude of the resistance only, but in general character. The adiabatic formula gives pressures which increase with the velocity much more rapidly than the pressures found by experiment, and I think the reason for this is that the formula credits the moving body with the energy required to pass through air at the maximum density with the prescribed velocity, whereas in the real case the density of the gas is the ordinary atmospheric density, and the body merely pushes a small cap of condensed air in front of it.

It must, I think, be admitted that some central part of this cap is raised to the full adiabatic pressure, but that part must extend over only a small fraction of the whole area, and that fraction itself must be a function of the velocity, and so that the law of resistance will be altered as well as the total amount. This is illustrated in fig. 5, where the curve A is the adiabatic resistance and B the resistance found by experiment; C represents $\frac{1}{2}\rho v^2$ (the

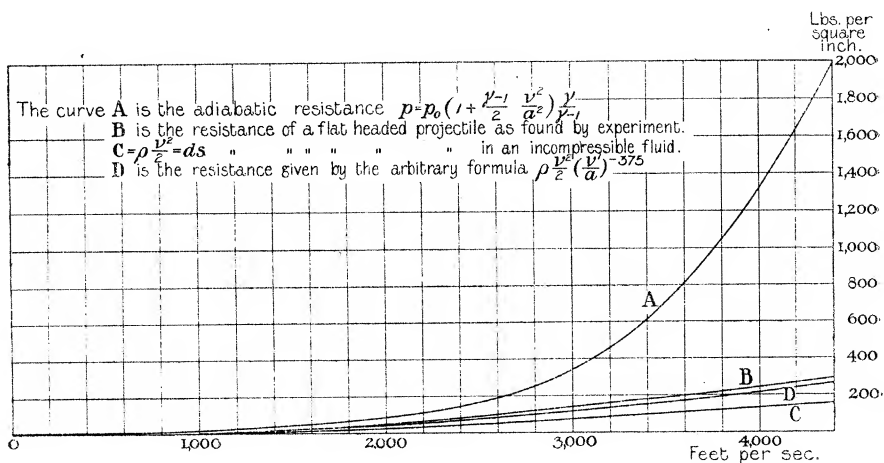


FIG. 5.

* 'Phil. Mag.,' Dec., 1876.

resistance which would be experienced per unit area in an incompressible fluid), and D is computed by the arbitrary formula $\frac{1}{2}\rho v^2 (v'/a)^{0.375}$.

In this latter formula, v' is the velocity with which a wave is propagated in the air immediately in front of the body and in the axis of the motion.

This velocity is the same as the velocity of the body when that exceeds the velocity of sound in the undisturbed gas, and it will be noticed that the pressure given by this formula, plus one atmosphere (on account of the defect pressure at the tail end), agrees well with experiment from velocities 2400 to 4500, beyond which I have not gone.

The diagram shows that the experimental resistance, although greater than $\frac{1}{2}\rho v^2$, is nearer to it than to the adiabatic pressure.

If we subtract $\frac{1}{2}\rho v^2 + p_0$ from the experimental value of the resistance, it is obvious that the difference (shown by the curve Z , fig. 9) is approximately a parabola whose ordinates are given by constant $X(v-a)^2$; and this element of resistance may be attributed to the force required to generate the waves which accompany the body when its velocity is greater than the velocity of sound.

The best evidence as to the nature of the waves in question may be drawn from the shadow photographs of rifle bullets taken by Mr. C. V. Boys.

One of these photographs (reproduced by permission in fig. 6) shows

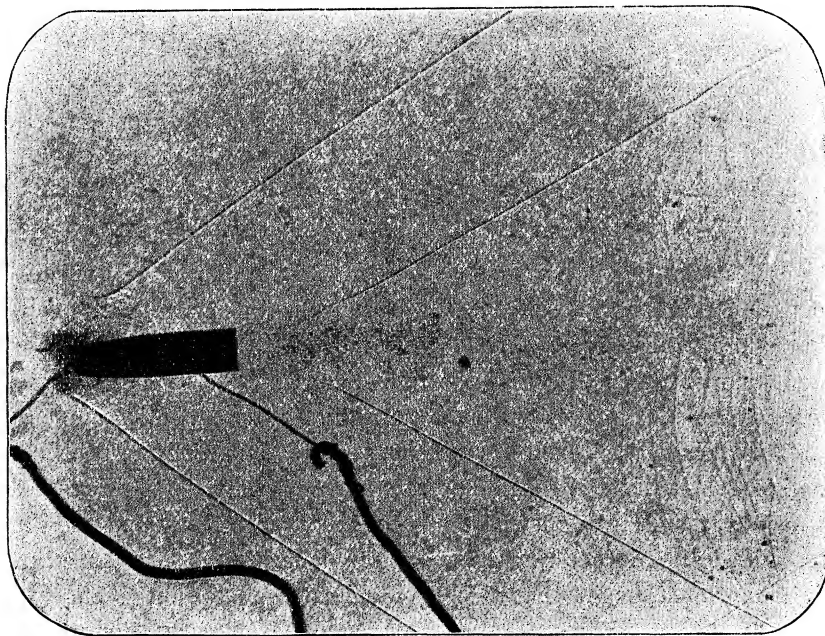


FIG. 6.—Photograph (by Professor C. V. Boys) of 0.303 rifle bullet at 2000 f.s.

a bullet travelling at about 2000 f.s. Accompanying it are two quasi-conical wave sheets having the same axis. The angle of the cone, of course, depends on the ratio of the velocity of the bullet to the velocity of sound, and the angle must be such that the velocity of the bullet resolved normal to the wave surface is equal to the velocity of sound in the gas in the condition as to pressure, density, etc., which obtains at the position through which the normal is drawn. The curvature, therefore, of the generating lines of the surfaces indicates that the conditions of the surrounding air are changed for a distance which is considerable, compared with the dimensions of the bullet.

The very sharp delineation of the wave in the photographs does not really imply an equally rapid change in the density at the wave surface, as may be seen from fig. 7, which is a shadow photograph of a heated tube.

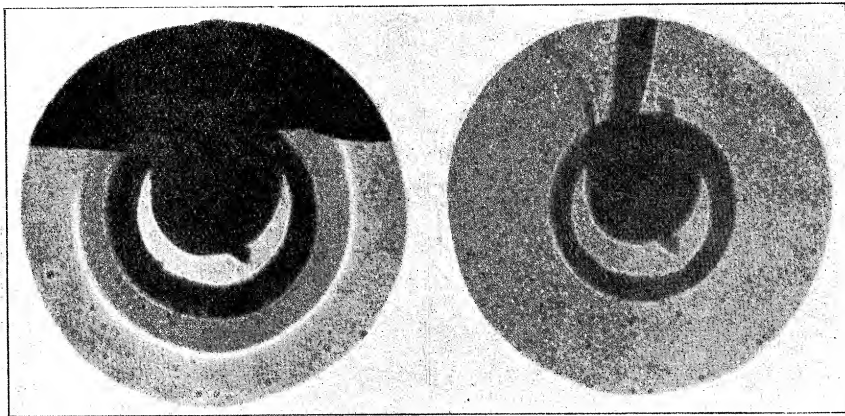


FIG. 7.

A small source of light was placed about 10 feet from the tube in the prolongation of its axis. The tube itself contained a small heating coil, and shadow photographs were taken at various distances with the tube raised to different temperatures. In fig. 7 the right-hand half of the picture is merely the geometrical shadow of the tube at ordinary atmospheric temperature. The left-hand half was taken in the same position but with the tube heated.

It will be seen that the warming of the air near the tube has deflected the light outwards, so as to somewhat magnify the shadow, and that the shadow is surrounded by a bright ring in which the deflected light is concentrated, but it cannot be thought that this ring indicates an abrupt change in the temperature of the air.

The varying temperature of the gas causes it to act on the beam of light

as a curved prism would, and to deflect the rays from the hotter to the cooler part of the air. In the analogous case of the bullet waves, the tendency is for the rays to be deflected towards the denser air.

Turning again to fig. 6, we see that both in the wave which precedes and in that which follows the bullet, the bright line is behind the dark one, showing that in both the most rapid change in the condition of the air is from a low to a higher density, that is, that both waves have steep fronts and that the pressure rises more quickly than it falls.

The origin of the wave in front of the bullet is obviously the compression of the air by the rapid motion, but it does not at first sight appear why a wave of compression should be set up well in its rear. This stern wave does not begin to show until the velocity is a good deal higher than the velocity of sound, and is caused by the closing together of the streams after they have passed the end of the bullet behind which, at such velocity, they leave a nearly vacuous space, plainly shown in all the photographs in which the stern wave appears. It will be noticed that the stern wave has its origin at the place where the wake is narrowest. An analogous phenomenon may be seen in water when a stream flows past an obstruction at such a speed as to leave a clear space behind it, the streams, as they meet again from either side, then mounting in a formidable-looking wave.

On the whole, therefore, it seems that the resistance of a body moving in a compressible gas may be represented as the sum of three terms, namely:—

- (1) The positive pressure on the head (X) (treating the fluid as incompressible).
- (2) The negative pressure on the tail (Y).
- (3) The force necessary to generate the waves (Z).

The work done by (1) or (2) remains as velocity in the fluid forming the wake, and the wave-making energy, of course, travels along into space with the wave.

X varies as the square of the velocity. Y depends, in the first instance, on the difference of pressure which must exist between two regions if air is to flow from one to the other with the velocity of the bullet; the higher pressure being in this case the pressure of the atmosphere, but, owing to the conditions in the wake, the simple expression for this difference does not apply.

The wave-making resistance (Z) depends on $(v-a)$, and is apparently proportional to the square of that quantity.

I am not aware that any dynamical solution has been found for the

three-dimensional problem concerning the motion which would be set up in an elastic gas by the passage through it of a small region in which a constant pressure was maintained differing from the general pressure in the gas, when that difference is not small. Earnshaw's investigations, however, regarding plane waves of finite amplitude may be applied to the case of the resistance experienced by a frictionless piston moving in an infinitely long tube filled with gas.

Earnshaw shows that where the velocity of the gas is v' the pressure is $p_0 e^{v/a}$, and this indicates that the piston will drive in front of it a pond of condensed air at pressure $p_0 e^{v/a}$ whose length, measured from the piston, increases at the rate $v_0 \rho_0 / \rho$, but of course the projectile cannot carry with it an ever-growing cap of condensed air.

Assuming that the resistance of a projectile can be represented by the sum of the three terms X, Y, and Z, we can determine the constants in X and Z for flat-headed projectiles from the experimental results at velocities over 2400 f.s.,* for above this speed Y is constant and equal to 1 atmosphere.

If there were reliable data for the resistance of flat-headed projectiles down to 1100 f.s., we could at once, knowing X and Z, determine also, so far, the form of Y; but such data regarding flat heads are wanting.† We can, however, get at Y indirectly, for the resistance of pointed projectiles is known with considerable exactness up to velocities of 3500 f.s., and by comparing the resistance of the pointed and flat-headed projectiles at velocities above 2400 f.s., the reduction in the coefficients in X and Z due to the pointed form can be found, and then Y, which is unaffected by the shape of the head, can be found from the experiments on pointed projectiles.

The behaviour of the ordinary cup anemometer shows that at moderate speeds the ratio of the head resistance of the convex, to that of the concave, hemisphere is of the order of 1/4, and the coefficient $\frac{1}{4}$ seems to apply fairly to the relation between the head resistance of pointed and flat-headed projectiles at high speeds.

Using $\frac{1}{4}$ as the coefficient for the resistance of the pointed head, I find that the wave-making coefficient must be multiplied by about 1/5, owing to the change of shape, and that in a pointed projectile the resistance is fairly well expressed in C.G.S. units by the equation

$$R = \frac{1}{4} \rho_0 \frac{1}{2} v^2 + (1 - A)^{0.354} + 1.35 \times 10^{-7} (v' - a)^2, \quad (1)$$

* 'Roy. Soc. Proc.,' Nov. 17, 1904, "Air Resistance."

† Most of the experiments on this subject have been made with the corner of the flat head rounded off, which leaves the effective area doubtful.

the corresponding equation for flat heads being

$$R = \rho_0 \frac{1}{2} v^2 + (1-A)^{0.354} + 7.5 \times 10^{-7} (v' - a)^2. \quad (2)$$

Here ρ_0 = the density of the undisturbed air.

p_0 = „ pressure „ „

$A = \left(1 - \frac{\gamma-1}{2} \frac{v^2}{a^2}\right)^{\gamma/\gamma-1}$, so that $1-A$ is the resistance.

v' = the velocity of sound in the air just in front of the projectile, and is equal to v when $v' > a$.

These curves are plotted in figs. 8 and 9 (pp. 272 and 273) respectively, and the corresponding experimental values are marked by dots.*

One of the most remarkable features of the resistance curve for pointed projectiles is, that for velocities ranging from that of sound up to 3000 f.s. it is almost a straight line which, if produced, would cut the axis of abscissæ at a point where the velocity is about 850 f.s.

In a future paper advantage will be taken of this to find a simple formula giving the distance which any shot will traverse in a given time.

To determine the resistance of the air at quite low velocities, experiments have been made by many investigators, including the late W. Froude (in 1870, about) and Professor Langley, but most recently by Dr. Stanton at the National Physical Laboratory.

The resistance, of course, varies as the square of the velocity, and the coefficients formed for v^2 in C.G.S. units vary from 8.68×10^{-7} (Froude) to 6.55×10^{-7} (Stanton).

I think it should be noticed that Dr. Stanton's coefficient is almost exactly $\frac{1}{2}\rho$. It is the lowest value found by any experimenter, too low in fact, since, if correct, it would indicate that there was scarcely any negative pressure on the down stream side of the planes he used.

Further experiments are desirable in which the pressure on the up and down-stream side shall be measured separately, and this both for high and low velocities. I have reason to believe that an apparatus might be made which, for a moderate expenditure of power (about 10 horse power), would supply a continuous stream of air about 6 inches in diameter flowing with a velocity of over 1000 feet per second, and although the current would probably be a turbulent one, I should not expect that the resistance experienced by a body placed in it would, if averaged for a few seconds, differ greatly from the resistance in a smoothly flowing stream of the same mean velocity.

* In the diagrams, the units employed are pounds per square inch and feet per second.

At velocities of this order, the resistance is of the order of pounds per square inch (something like 10 lbs. per square inch for a plane at the velocity of sound), hence very measurable forces would be exerted on bodies.

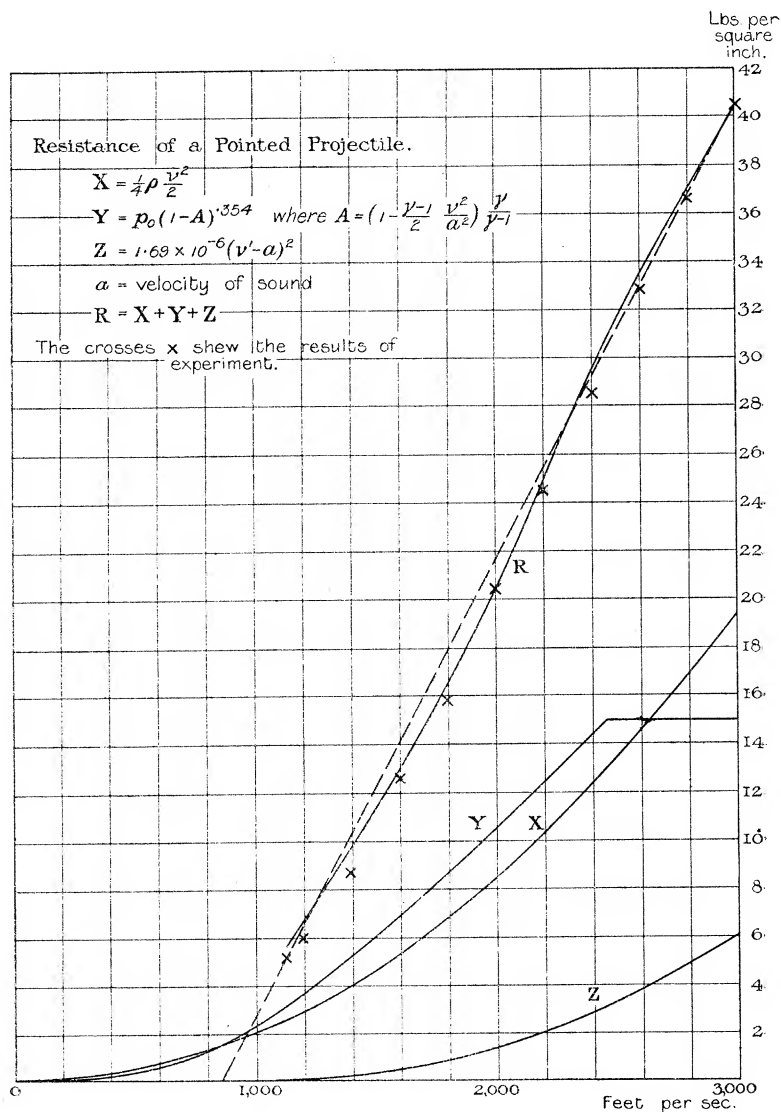


FIG. 8.

small compared to the diameter of the stream, and therefore not influenced by the walls which confine it. Much interesting information might be obtained from experiments of this nature.

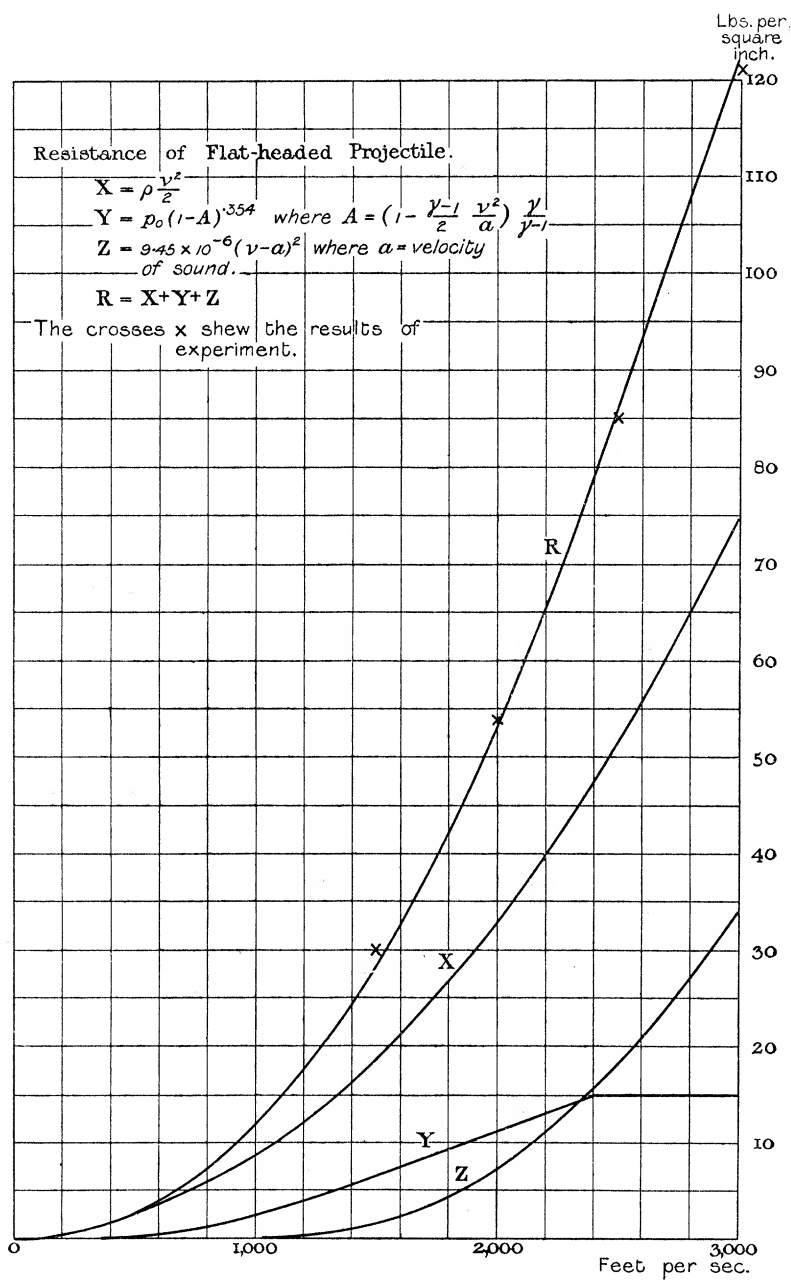


FIG. 9.

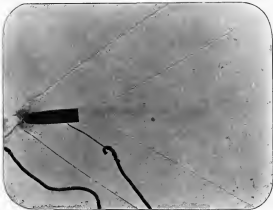


FIG. 6.—Photograph (by Professor C. V. Boys) of 0.303 rifle bullet at 2000 ft.

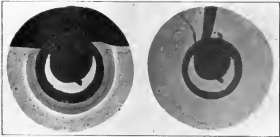


FIG. 7.